Macroscopic Chromodynamics of a Confining Medium

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Abstract

A nonminimal interaction of the gluon field with a massive antisymmetric tensor order parameter is shown to yield a vanishing dielectric function at tree level. The London-type description of this perfect diaelectric suggests a tractable non-Abelian chromodynamics of the Ginzburg-Landau type which is suitable for describing the confinement-deconfinement phase transition.

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We still lack a microscopic understanding, in terms of QCD, of the experimental fact that colored quarks and gluons do not escape separately from colorless hadrons. The confinement of color can however be understood if the vacuum is assumed to exhibit the property of a dual Meissner effect, that is,

if it behaves as a perfect color diaelectric [1],[2],[3],[4]. This letter presents a phenomenological London-Ginzburg-Landau type model exhibiting *bona fide* this property. The model provides for an operational framework suitable for the description of various properties of the confinement-deconfinement phase transition at finite temperature and in strong chromoelectric fields. We hope that our phenomenological model may ultimately serve as a guide towards a more microscopic description.

The basic question is: which order parameter can produce a gauge invariant interaction with the gluon field A_a^μ such that the color dielectric function ϵ vanishes in the vacuum at tree level? Consider first the following bilinear (i.e. London type) Lagrangian:

$$\mathcal{L}_L = -\frac{1}{4}(\partial_\mu A_{a\nu} - \partial_\nu A_{a\mu})^2 - \frac{1}{8}(\epsilon^{\mu\nu\alpha\beta}\partial_\nu \Phi_{a\alpha\beta})(\epsilon_{\mu\eta\sigma\tau}\partial^\eta \Phi_a^{\sigma\tau})$$

$$-\frac{1}{4}f_0^2 \Phi_{a\mu\nu}\Phi_{a\mu\nu} + \frac{1}{2}g_0 \Phi_{a\mu\nu}(\partial_\mu A_{a\nu} - \partial_\nu A_{a\mu}) \tag{1}$$

which possesses Abelianized gauge invariance $A_{a\mu} \rightarrow A_{a\mu} + \partial_\mu \alpha_a$. The Lagrangian (1) is intended to form the basis for a truly interacting Ginzburg-Landau field system. The order parameter is provided by the antisymmetric tensor $\Phi_{a\mu\nu} = -\Phi_{a\nu\mu}$. It describes positive energy spin-1 excitations [5] with mass f_0 which correspond to three independent propagating space-space components Φ_{amn}. The constants $f_0^2 \geq g_0^2$ characterize the ordered (confining) phase.

The polarization tensor $\Pi_{\mu\nu}(k) \equiv (k_\mu k_\nu - k^2 g_{\mu\nu})\,\Pi(k^2)$ of the gluon field relates the bare gluon propagator $D_{\mu\nu}(k) \equiv (-g_{\mu\nu} + k_\mu k_\nu / k^2)/k^2$ in Landau gauge to the full propagator $\Delta_{\mu\nu}$, in terms of the dielectric function $\epsilon(k^2)$:

$$\Delta_{\mu\nu} = \frac{D_{\mu\nu}}{1 + \Pi(k^2)} \equiv \frac{D_{\mu\nu}}{\epsilon(k^2)} \tag{2}$$

The propagator of the Φ field, deduced from (1) is:

$$D_{\mu\nu;\rho\sigma}(k) = \frac{1}{k^2 - f_0^2}(G_{\mu\nu;\rho\sigma} - \frac{1}{f_0^2}K_{\mu\nu;\rho\sigma}) \tag{3}$$

where $G_{\mu\nu;\rho\sigma} \equiv g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}$ and $K_{\mu\nu;\rho\sigma} \equiv k_\mu k_\rho g_{\nu\sigma} - k_\nu k_\rho g_{\mu\sigma} - k_\mu k_\sigma g_{\nu\rho} + k_\nu k_\sigma g_{\mu\rho}$. We can use the propagator eq.(3) and the interaction term in (1)

to derive thus the following expression for the polarization tensor, in the tree approximation:

$$\Pi_{\nu\sigma}(k) = -g_0^2 k^\mu k^\rho D_{\mu\nu;\rho\sigma}(k) \tag{4}$$

It follows that $\Pi(k^2) = -g_0^2/f_0^2$ so that the dielectric function is equal to:

$$\epsilon\left(k^2\right) = \frac{f_0^2 - g_0^2}{f_0^2} \tag{5}$$

We see that if $f_0 = g_0$, the dielectric function vanishes and the corresponding gluon propagator becomes infinite. By definition [4], the simple Lagrangian (1) describes the medium outside the hadronic bags. Due to Lorentz invariance, which implies $\epsilon\mu = 1$, where μ is the magnetic permeability, the vacuum behaves simultaneously as a perfect paramagnet.

In fact, when $f_0 = g_0$, the Lagrangian (1) can be rewritten in the form [6]:

$$\mathcal{L}_L = -\frac{1}{8}(\epsilon^{\mu\nu\alpha\beta}\partial_\nu\Phi_{a\alpha\beta})(\epsilon_{\mu\eta\sigma\tau}\partial^\eta\Phi_a^{\sigma\tau}) - \frac{1}{4}(\partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} - g_0\Phi_{a\mu\nu})^2 \tag{6}$$

It may be checked that the Lagrangian (6) depends only on the field $(\partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} - g_0\Phi_{a\mu\nu})$ so that the gluon field is effectively absorbed by the Φ field. The Lagrangian (6) exhibits a new type of gauge invariance:

$$\Phi_{a\mu\nu} \to \Phi_{a\mu\nu} + \partial_\mu\xi_{a\nu} - \partial_\nu\xi_{a\mu} \qquad A_{a\mu} \to A_{a\mu} + g_0\xi_{a\mu} \tag{7}$$

besides the original (Abelianized) gauge invariance $A_{a\mu} \to A_{a\mu} + \partial_\mu\alpha_a$. Consequently, the massless gauge field $A_{a\mu}$ may be viewed as simply modifying the gauge of the Φ field.

If $g_0 = 0$ in the London Lagrangian (1) the gauge field $A_{a\mu}$ decouples from the massive order parameter $\Phi_{a\mu\nu}$ and the dielectric function $\epsilon = 1$. This is the regime we expect to prevail inside a hadronic bag.

To achieve dynamically the transition between the vacuum phase $g_0 = f_0$ outside the bag and the phase $g_0 = 0$ inside, we introduce a real colorless scalar field $\chi(x)$ which develops a condensate in the vacuum due to a potential

$$V(\chi) = \frac{1}{2}m^2\chi^2 - \frac{1}{3}m\xi\chi^3 + \frac{1}{4}\lambda\chi^4 \qquad (m, \xi, \lambda > 0) \tag{8}$$

For $\xi^2 > 9\lambda/2$ the potential $V(\chi)$ has the essential feature of presenting a minimum at a nonzero value $\chi_0 \equiv M$:

$$\chi_0 \equiv M = \frac{1}{2} m \frac{\xi}{\lambda} \left(1 + \sqrt{1 - 4\lambda/\xi^2} \right) \tag{9}$$

The coupling between the scalar field χ and the color fields is achieved by replacing the constants f_0 and g_0 respectively by the functions $f(\chi)$ and $g(\chi)$ which we choose to be the lowest order polynomials $g^2(\chi) = (\alpha - 1)\chi^2$ and $f^2(\chi) = (\chi - M)^2 + (\alpha - 1)\chi^2$. This Ginzburg-Landau type extension of the London Lagrangian (1) is phenomenological and must be confronted to experiment. Renormalizability, unlike in the Higgs case, is not a guiding principle in such an effective low momentum theory. Thus an $SU(3)_c$ gauge invariant Ginzburg-Landau type Lagrangian which reduces to (1) for small $\Phi_a^{\mu\nu}$ and A_a^μ and for $\chi \to M$ is:

$$\mathcal{L}_{GL} = -\frac{1}{4} F_{a\mu\nu} F_a^{\mu\nu} - \frac{1}{8} (\epsilon^{\mu\nu\alpha\beta} D_\nu \Phi_{a\alpha\beta})(\epsilon_{\mu\eta\sigma\tau} D^\eta \Phi_a^{\sigma\tau})$$

$$- \frac{1}{4} f^2(\chi) \Phi_{a\mu\nu} \Phi_a^{\mu\nu} + \frac{1}{2} g(\chi) \Phi_{a\mu\nu} F_a^{\mu\nu} + \frac{1}{2} (\partial_\mu \chi)(\partial^\mu \chi) - V(\chi) \tag{10}$$

Two limiting cases corresponding to a constant χ field are of interest. First, for a small gauge field, the equilibrium χ is determined by solving $V'(\chi) = 0$ and the equation of motion of the $\Phi_{a\mu\nu}$ field may be used to eliminate it in favor of the gluon field, yielding:

$$\Phi_{a\mu\nu} = \frac{g(\chi)}{f^2(\chi)} F_{a\mu\nu} \tag{11}$$

The Abelian version of the Lagrangian (10) reduces then to [3]:

$$\mathcal{L}_{eff} = -\frac{1}{4} (1 - \frac{g^2(\chi)}{f^2(\chi)}) F_{a\mu\nu} F_a^{\mu\nu} + \frac{1}{2} (\partial_\mu \chi)(\partial^\mu \chi) - V(\chi) \tag{12}$$

and the dielectric constant becomes:

$$\epsilon(\chi) = \frac{f^2(\chi) - g^2(\chi)}{f^2(\chi)} = \frac{(\chi - M)^2}{(\chi - M)^2 + (\alpha - 1)\chi^2} \tag{13}$$

It vanishes, as it should, in the vacuum ordered phase $\chi = M$.

Second, for a strong chromoelectric field, the equilibrium value of χ is determined by solving [2]:

$$V'_{eff}\left(\chi, E_c\right) = \frac{d}{d\chi}\left(V\left(\chi\right) + \frac{g^2\left(\chi\right)}{f^2\left(\chi\right)}\frac{1}{2}E_c^2\right) = 0 \tag{14}$$

The term added to $V(\chi)$ increases with E_c. For a strong enough chromoelectric field $E_c > E_c^{crit}$, the minimum of the effective potential $V_{eff}(\chi, E_c)$ will shift to $\chi = 0$ thereby destroying the ordered phase.

At finite temperatures, the Ginzburg-Landau theory is not a standard finite-temperature theory. The use of the Lagrangian (10) at finite temperature requires an understanding of how its phenomenological parameters depend on temperature. Notice that if M decreases with temperature there should appear a temperatureT_c at which the condensate χ disappears and the system returns then to the perturbative phase with $\epsilon = 1$.

We emphasize that the behavior of the gluon field in the presence of the condensate $\chi = M$, which does not break any symmetry, is characteristic of neither the perturbative nor the Higgs phase. Referring to t'Hooft's phrasing [7] we believe that the Lagrangian (10) with the condensate $\chi = M$ is an example of a realization of the color confined phase.

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